A more efficient approach to perform sensitivity analyses in 0D/1D cardiovascular models

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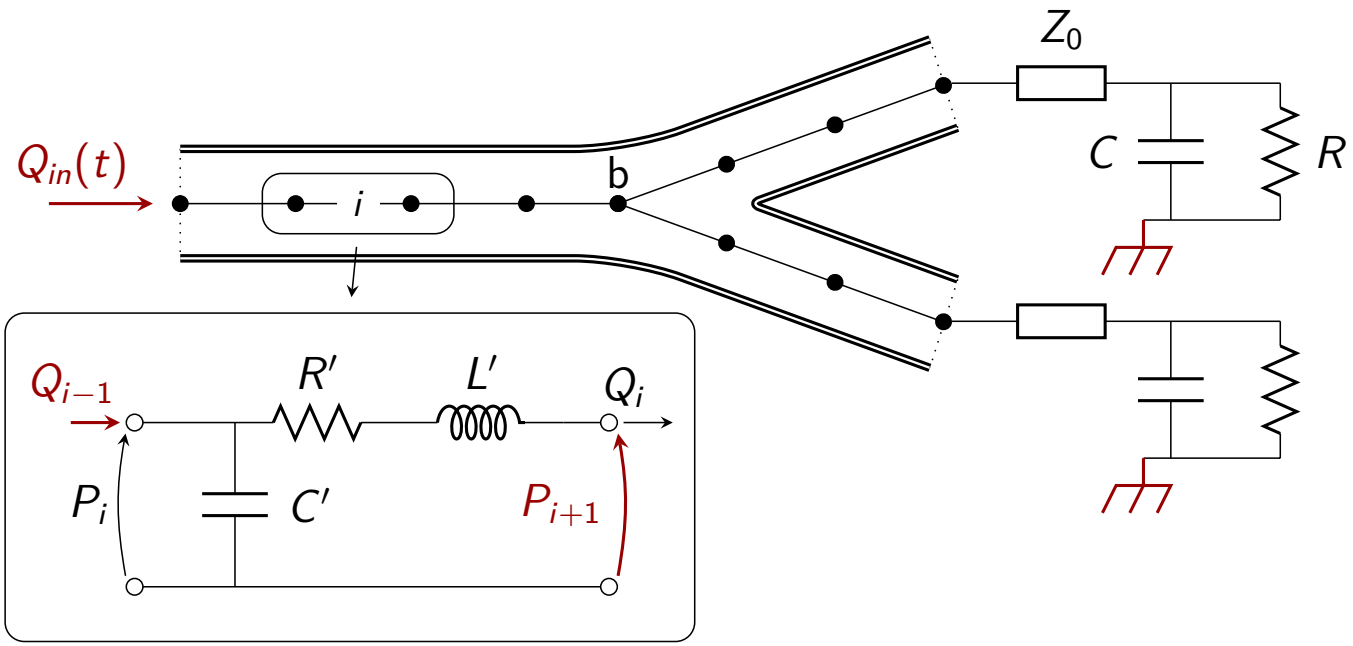
Introduction and context

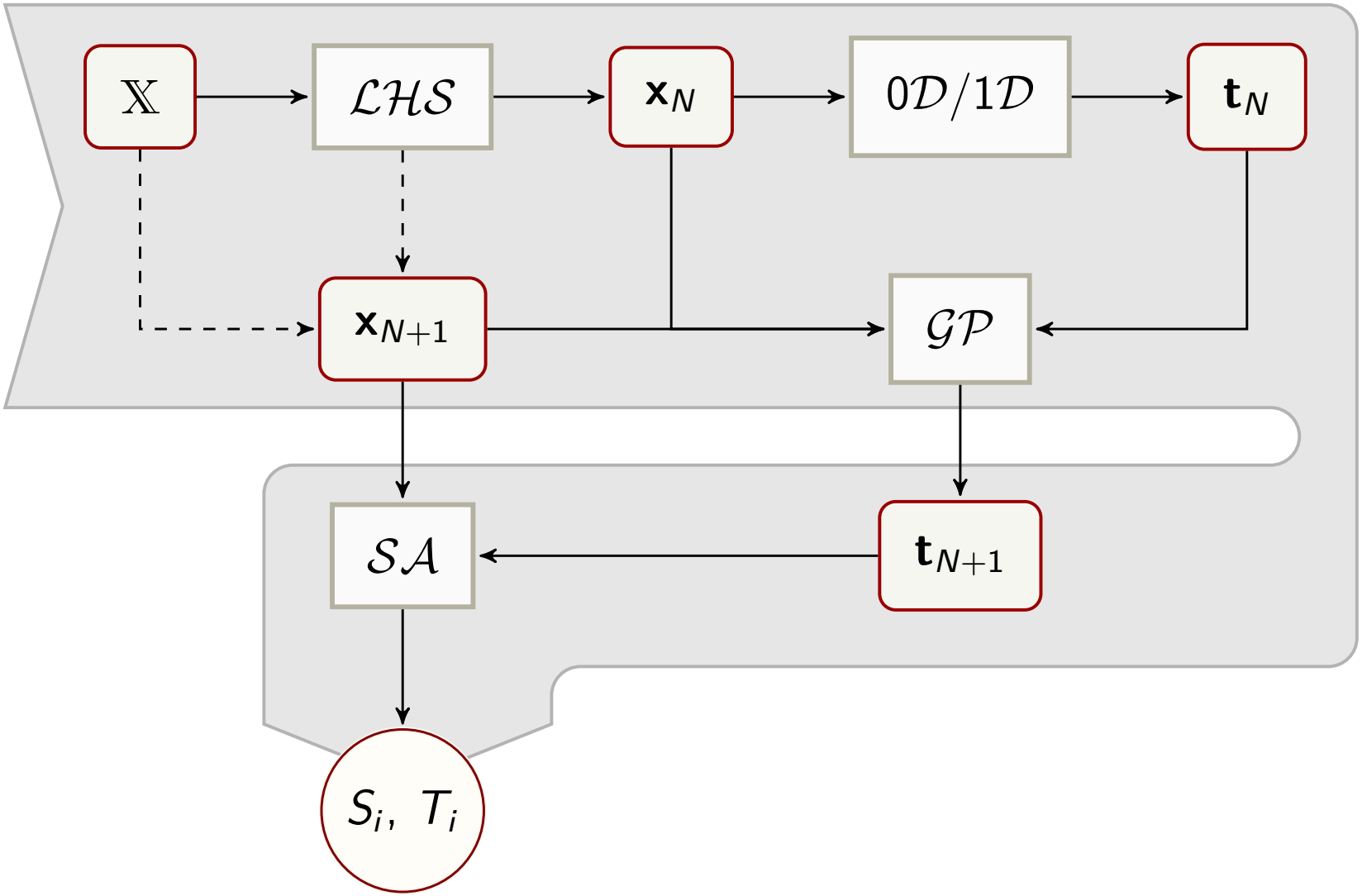
- PhD aim: to pulse wave propagation with aging via numerical methods
- 0D/1D models based on Navier-Stokes equations:
  - describe pulse wave propagation
  - huge number of parameters
- Sensitivity analysis $\rightarrow$ parameter prioritisation and parameter fixing
- Statistical emulator $\rightarrow$ make the most of few simulations

Symmetric bifurcation reduced model

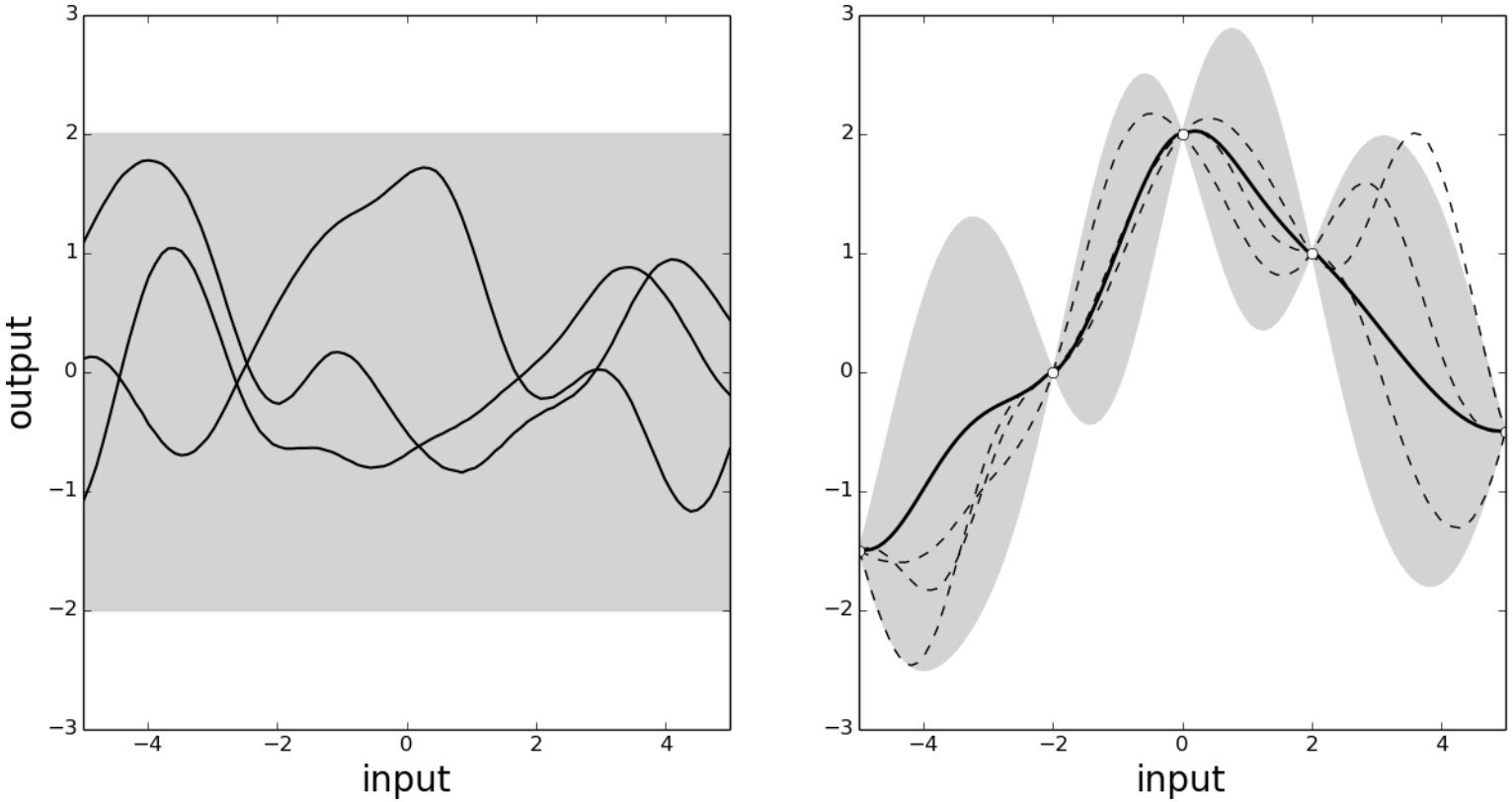


- First order Euler's method
- Conservation of mass and total pressure at b
- Pulse wave velocity as aging biomarker

Workflow


$\mathbb{X}$
$\mathcal{LHS}$
$\mathbf{x}_N$
$0\mathcal{D}/1\mathcal{D}$
$\mathbf{t}_N$
$\mathbf{x}_{N+1}$
$\mathcal{GP}$
$\mathcal{SA}$
$\mathbf{t}_{N+1}$
$S_i,\ T_i$

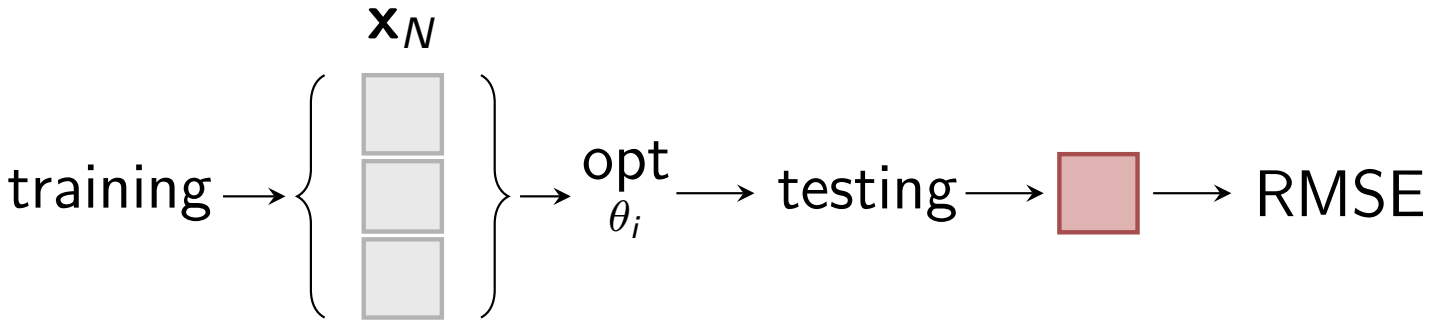
Gaussian Process



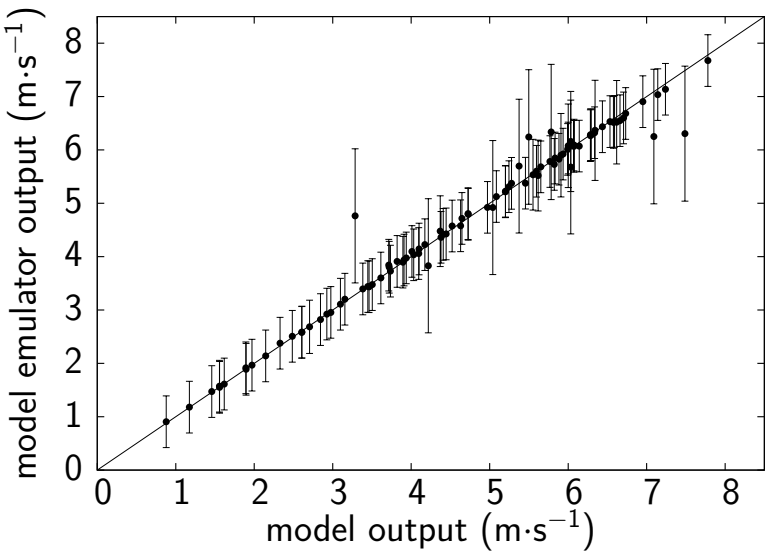
(Left) Three samples from prior distribution centred at zero and twice the standard deviation (shaded area). (Right) Posterior distribution trained on five data points (white). Mean prediction (solid) and three samples from prior distribution (dashed).

$$f(\mathbf{x}) \sim \mathcal{GP}(\mathbb{E}, \Sigma)$$

Gaussian Process optimisation & validation



- emulator outputs against model outputs with 95% confidence bars



Gaussian Process training

Parameter	*Range*
Blood viscosity μ (cP)	1–4
Blood density ρ (kg·m^{-3})	0.7–1.3
Vessel length ℓ (m)	0.2–0.8
Lumen radius r (m)	0.005–0.040
Wall thickness h (m)	0.001–0.005
Young's modulus E (kPa)	30–13000
Peripheral resistance R_s (g·cm^{-4}s^{-1})	1000–2000

- 100 samples $\rightarrow$ 32.6 hrs ($\sim$43 days)

Gaussian Process results

- 3200 samples $\rightarrow$ 1043.2 hrs
- $\mathcal{GP}$ running time $\sim 2ms$

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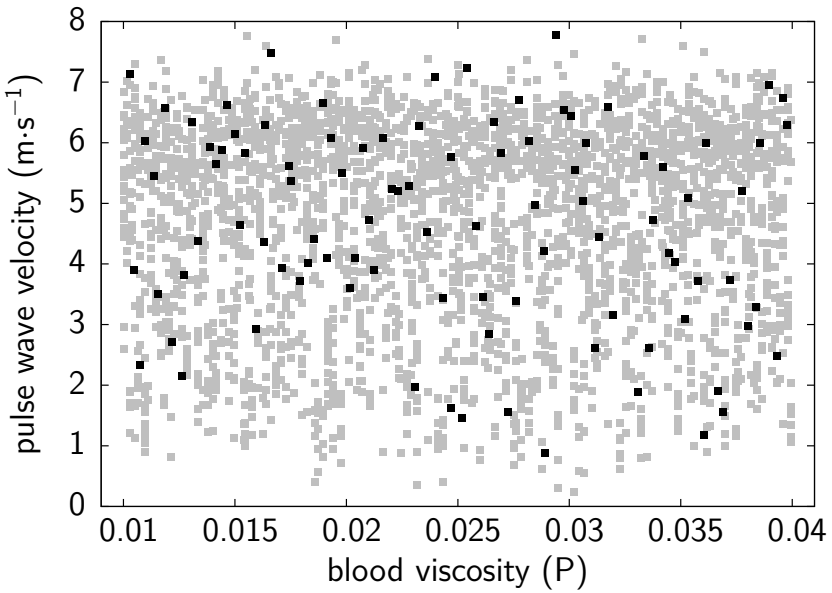
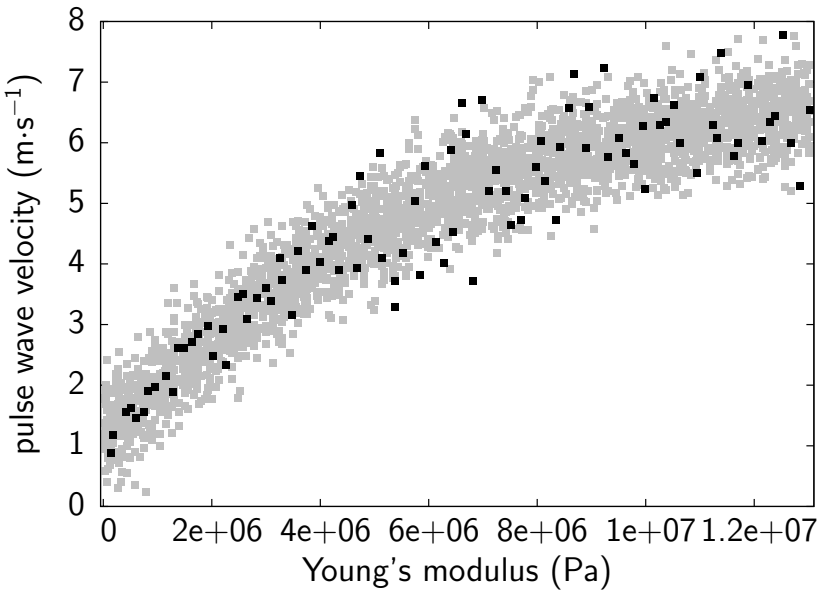




Emulator outputs (grey) and model training runs (black) against blood dynamic viscosity (left) and vessel wall Young's modulus (right).

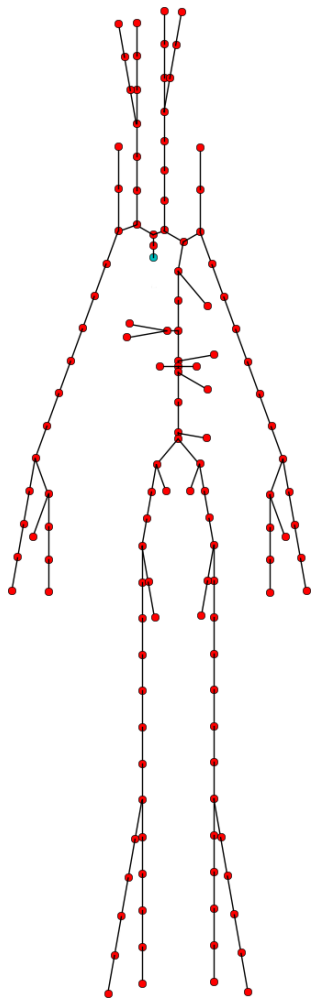
Sensitivity Analysis results

Parameter	S	T
Blood viscosity μ	0.049	0.052
Blood density ρ	0.098	0.051
Vessel length ℓ	0.018	0.048
Lumen radius r	0.024	0.046
Wall thickness h	0.032	0.051
Young's modulus E	0.770	0.707
Peripheral resistance R_s	0.018	0.045

Conclusions and future works

- 0D/1D models $\rightarrow$ pulse wave propagation
- $\mathcal{GP}$ + SA $\rightarrow$ computationally efficient

  2 ms vs 43 days
- SA complete model $\rightarrow$ aging



Thank you!

Sensitivity indices

Variance measures *how far each number* $y(\mathbf{x})$ *in the set is from the mean* y_0:

$$V = \int_{\Omega^d} y^2\left(x_1, x_2, ..., x_d\right) dx_1 dx_2 ... dx_d - y_0^2$$

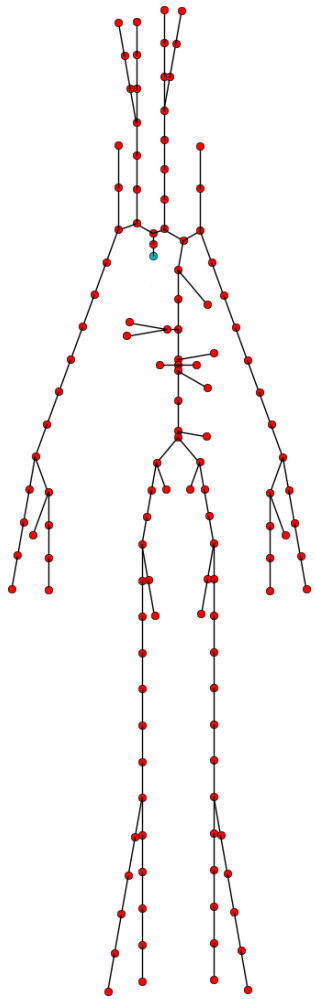
$$\overset{\text{ANOVA}}{=} \sum_{i=1}^{d} V_i + \sum_{1 \le i \le j \le d} V_{ij} + ... + V_{1,2,...,d}$$

$$S_i = \frac{V_i}{V} \qquad S_{ij} = \frac{V_{ij}}{V} \qquad ...$$

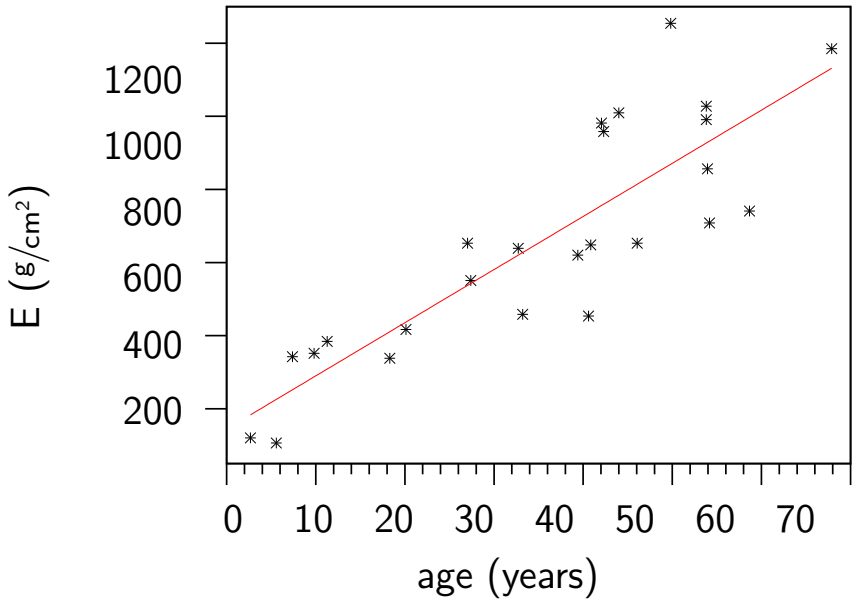
$$T_i = S_i + \sum_{j>i} S_{ij} + \sum_{j<i} S_{ji} + ... + S_{1,2,...,d}$$

Boris (work in progress)

- 1D finite volume solver
- bifurcations, anastomosis, conjunctions
- viscoelastic constitutive equation
- convective acceleration term
- full systemic arterial circulation
- heart model
- patient specific



Ageing parameters



- wall properties (Gozna, 1974)
- vessel geometry (Nichols, 1985)
- blood properties (Ajmani, 2000)