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# Arterial Haemodynamics

MEC434 - Biomechanics of the Cardiovascular System

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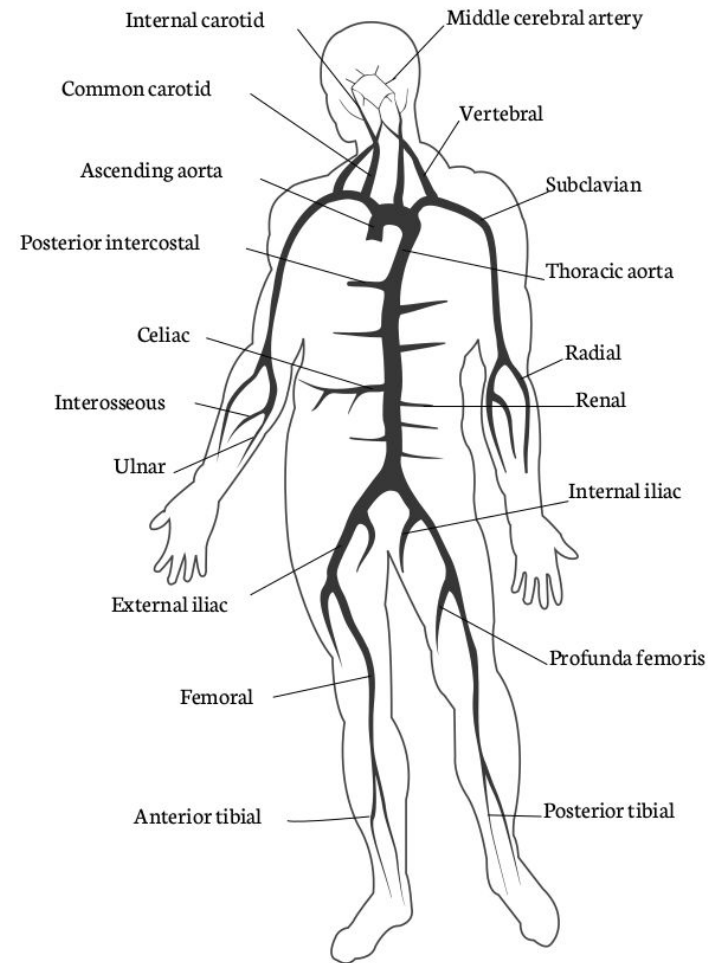
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INSIGNEO Institute for *in silico* medicine  
PLB E09

# Outline

- Cardiovascular system
- Arterial wall and compliance
- Incident wave
- Wave speed
- Reflection and transmission
- Superposition
- Augmentation index
- Waveform analysis

# Cardiovascular system

- The cardiovascular system comprises the heart, the two circulations - systemic and pulmonary - and the blood.
- The mechanical properties of the vascular wall change along the arterial tree: large vessels are elastic and compliant whereas peripheral arteries are narrow and stiff.



# Heart

- The blood is pumped through the cardiovascular system by the cyclic contraction of the heart.
- The pulsatile regime generates waves that travel down the aorta and major conduit arteries.
- These waves may be pressure waves, flow or velocity waves or even diameter waves.

atria + ventricles  
relax + fill

atria contract

isovolumetric  
contraction

ventricles  
contract;  
ejection

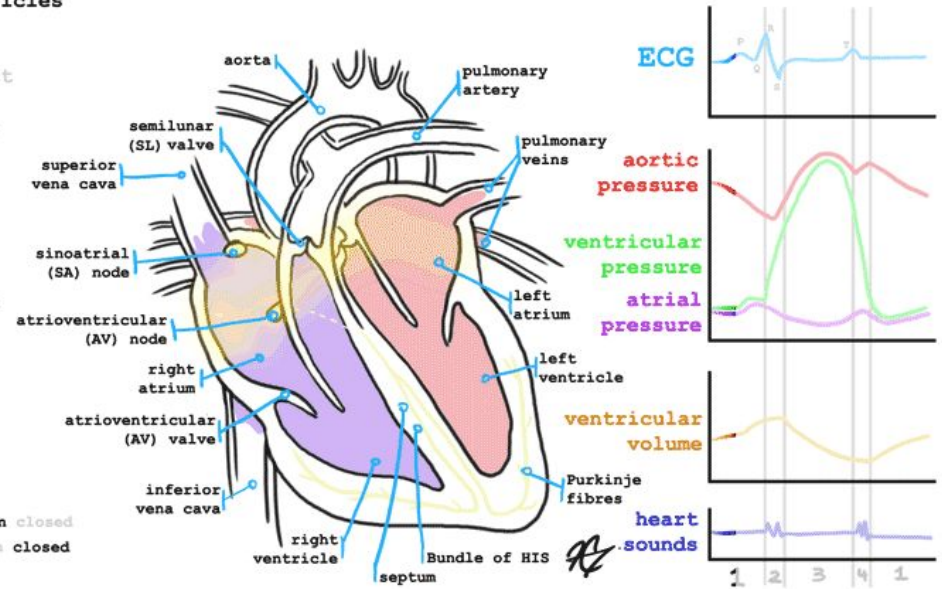
isovolumetric  
relaxation

Time: 0.1s

diastole  
systole

AV valves open closed

SL valves open closed



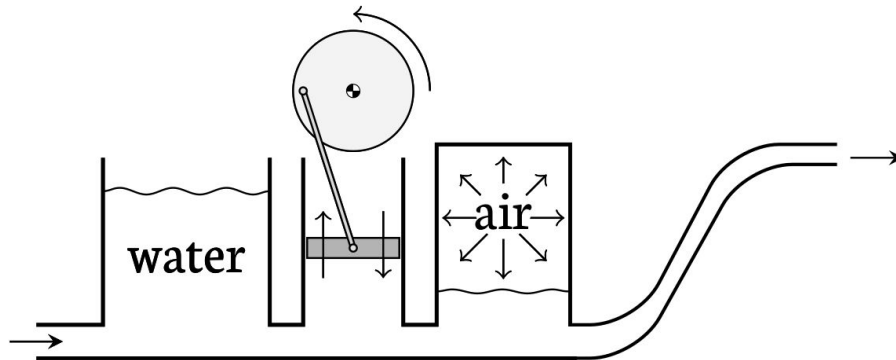
# Capillaries

- Capillary flow must be continuous (no pulsatility) and slow enough to allow the needed time for exchange of nutrients, gases, and waste.
- The arterial elasticity is responsible for converting the pulsatile flow exiting the heart in a constant and slow stream within the capillaries.
- 1899 Otto Frank, German physiologist: first mathematical description of the *windkessel effect*



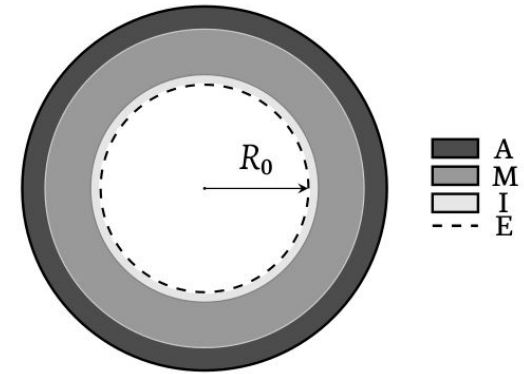
# Windkessel effect

- Windkessel = air chamber in German
- In a fire engine, the water is driven by a rotary pump and it is constantly sprinkled throughout the hose due to the air expansion chamber.
- The chamber accommodates a varying volume of water depending on the pump piston position. The outflow from the chamber is driven by the air inside the chamber that is compressed and slowly expands when the inflow from the pump stops.



# Arterial wall

- Arteries have strong muscular walls which help to maintain a tubular shape even in absence of flow.
- From large arteries to capillaries, along with the lumen radius drop, there is a stiffening of the walls. Large arteries contain a higher percentage of elastin and collagen than smaller vessels.
- The vessel *compliance* ( $C_A$ ) is the ability of vessel walls to deform and accommodate a variable volume of blood.

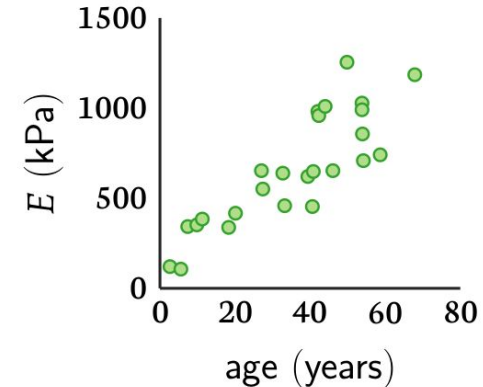
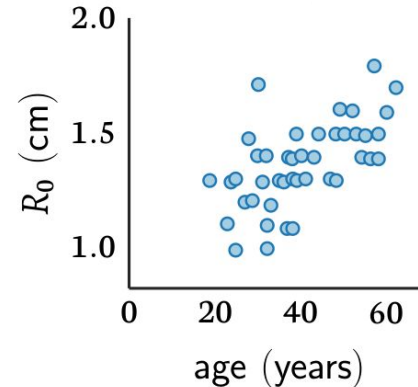


- *Adventitia*: collagen and elastin
- *Media*: thick layer of muscle cells and elastin
- *Intima*: thin elastin layer
- *Endothelium*: single layer of cells surrounded by elastin and collagen fibres

$$C_A = \frac{\Delta A}{\Delta P}$$

# Aging effect

- Structural changes due to ageing are spread uniformly over the entire arterial tree and concern primarily vessel local properties rather than the global topology.
- There is a general increase in vessel radius, in media and intima thickness, and in overall Young's modulus.





# Incident wave

- The fluid (blood) displaced by the piston (heart) flows into the elastic tube (aorta) distending it.
- The fluid continues to flow into the tube distending the tube more and more. The change in velocity and pressure in the tube generates a forward wavefront that propagates along the tube
- The piston is decelerated back to zero velocity and the velocity of the fluid moving into the tube decreases. This change in velocity and pressure causes a forward travelling decompression wave.
- The piston has stopped moving so that there is no new fluid moving into the tube. The two wavefronts continue to propagate along the tube and produce a wave that keeps propagating along the tube.



# Wave speed

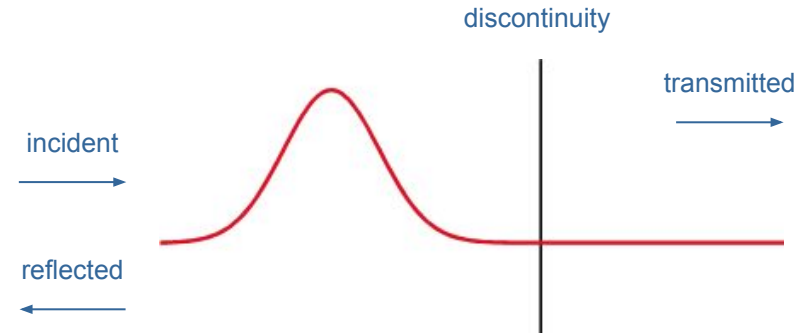
- Because of the elasticity of the major arteries, the waves are not transmitted instantaneously to the periphery, but they propagate at a certain speed.
- The pulse wave speed is related to the elastic modulus of the wall material via the Moens-Korteweg equation.
- In the aorta of a healthy subject  $c$  is typically  $4\text{-}5\text{ m/s}$ . In stiff aortas, having a high  $E$ , the wave speed is higher.
- The wave speed in peripheral arteries is higher than in central arteries since the elastic modulus is higher and diameter is small.

$$c = \sqrt{\frac{hE}{2R_0\rho}}$$

$$c = \sqrt{\frac{A}{\rho C_A}}$$

# Wave reflection and transmission

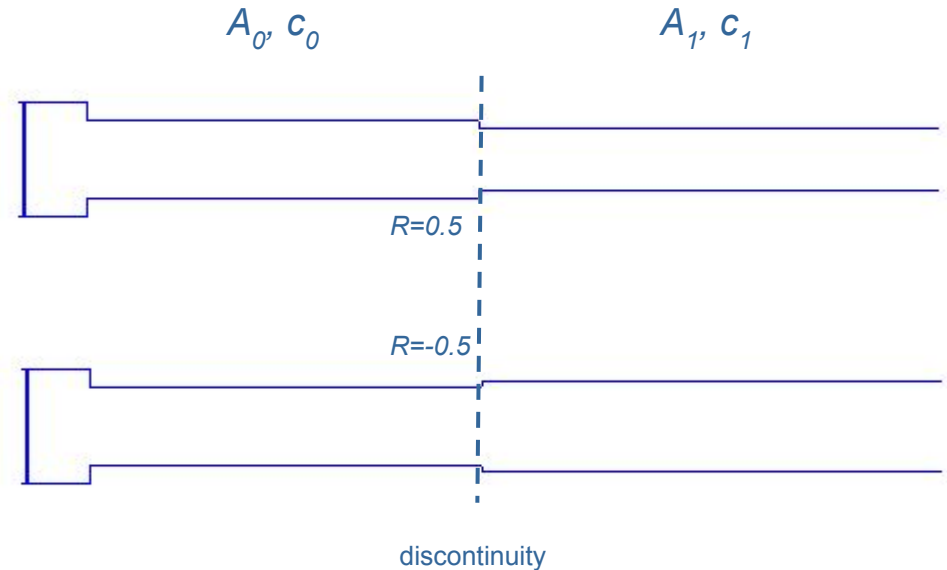
- Wave reflection takes place at all discontinuities of the vasculature (bifurcations, stenosis, stiffenings)
- The moment the reflected wave returns at the heart depends on the distance of the reflection site, the wave speed, and how the wave is reflected.



# Reflection coefficient

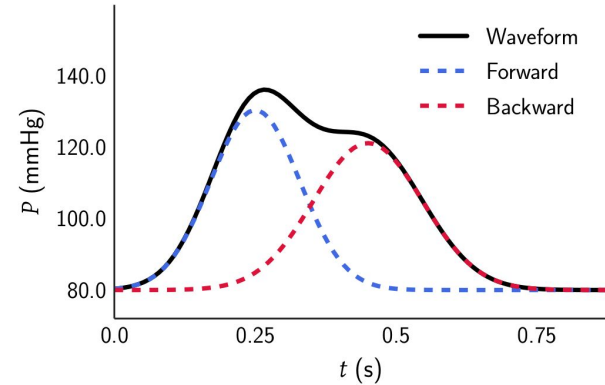
- Any discontinuity in the properties of the artery will cause the wavefronts to produce reflected and transmitted waves according to the type of discontinuity.
- The conservation of mass and energy at a discontinuity in an elastic vessel require that an incident wave with a pressure change  $\Delta P$  must generate a reflected wave with pressure change that is given by a *reflection coefficient*

$$R = \frac{\frac{A_0}{c_0} - \frac{A_1}{c_1}}{\frac{A_0}{c_0} + \frac{A_1}{c_1}}$$



# Waves superposition

- Wave pulses do not bounce off of each other, instead, they pass right through each other only showing any effect when they are overlapping. When the two pulses overlap, the resulting wave is the algebraic sum of the two pulses. This is known as the principle of superposition.
- Reflected waves superpose on the incident wave from the heart.



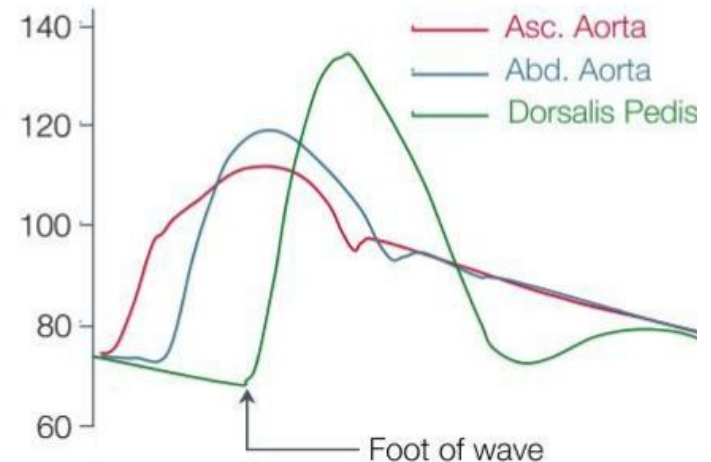
Meliss A. (2017)



Tom Walsh <http://www.ophysics.com/>

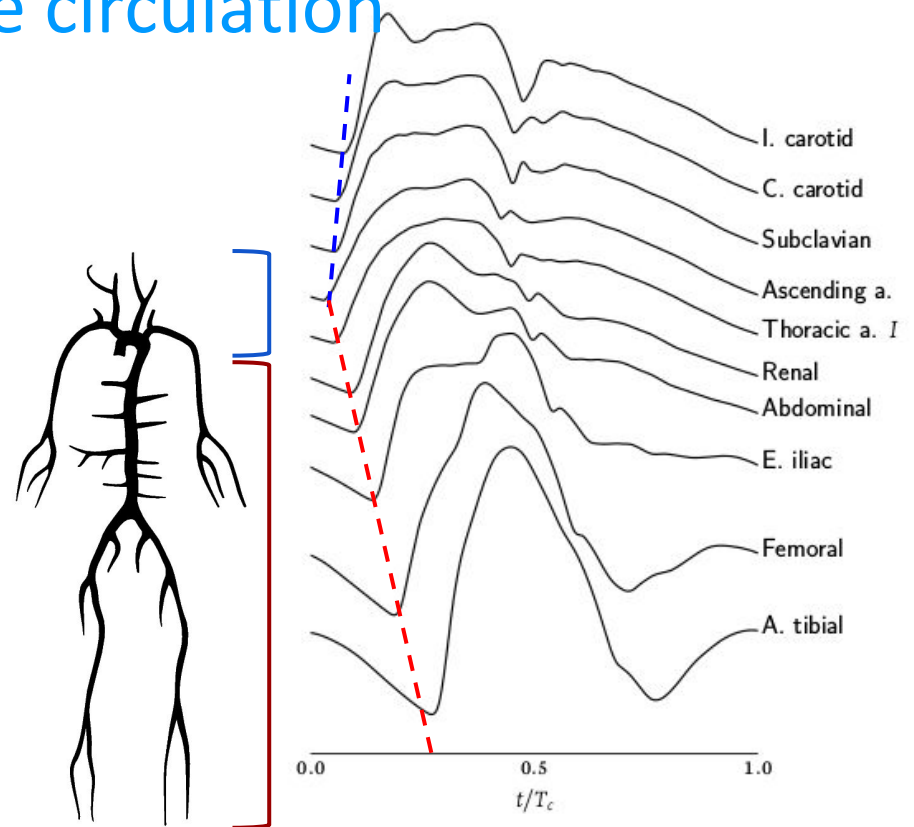
# Foot-to-foot measurement

- The ratio of the distance,  $\Delta x$ , and the time it takes for the wave to travel over this distance,  $\Delta t$ ,  $\Delta x/\Delta t$ , gives the pulse wave velocity,  $c$ .
- Wave speed can be estimated from the time it takes for the foot pressure to travel between two sites a known distance apart.
- The foot-to-foot wave velocity is an indirect measurement of arterial stiffness.



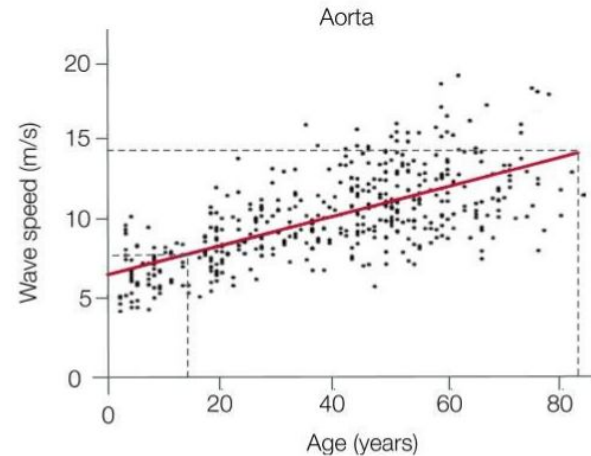
# Wave reflection across the circulation

- Pressures in the systemic circulation along a single cardiac cycle.
- The foot of the wave arrives later in the periphery and the reflected wave returns later to the heart.



# Wave speed and age

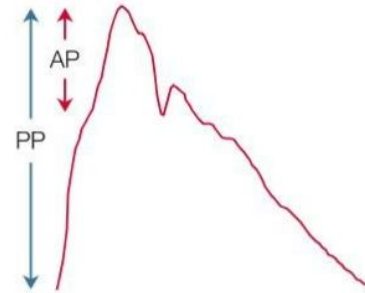
- Wave speed increases with age
- The increase in aortic stiffness with age is primarily attributed to a progressive thinning, fraying and fracture of elastic laminae
- This is due to repetitive cyclic stress of the pulsing pressure
- The stiffer arta implies reduced windkessel function and higher pulse pressure





# Augmentation index

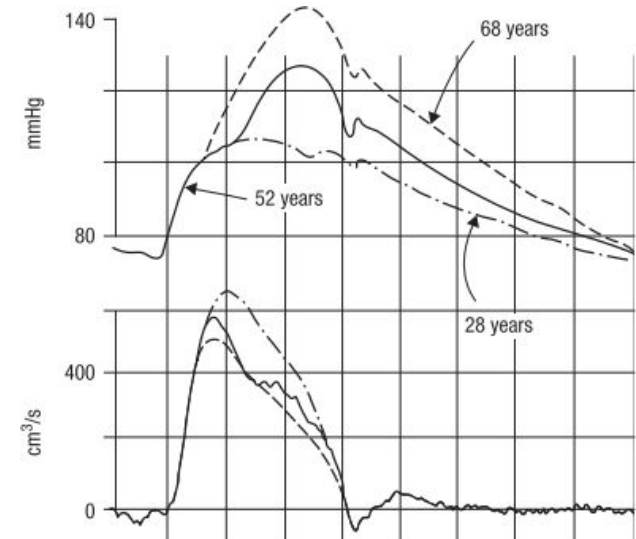
- The amount of reflection is related to the augmentation index, (AI).
- The AI is the ratio between the augmentation pressure (AP) and the pulse pressure (PP).
- The AI is a biomechanical marker commonly used in clinical practice



$$AI = \frac{AP}{PP}$$

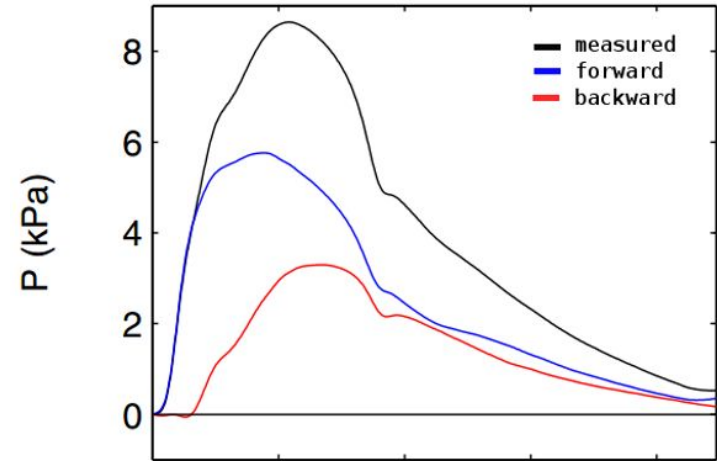
# Clinical context

- In hypertension, wave speed is increased. This results in large reflected pressure wave arriving at the heart in systole, adding to the forward wave, resulting in a large systolic pressure.
- With age the wave speed increases and reflected waves return earlier.



# Waveform analysis

- The reflected and incident pressure (and flow) waves are related by the characteristic impedance  $Z_c$ .
- The formulas for  $P_f$  and  $P_b$  can be used when  $Z_c$  is a real number (inviscid blood and elastic walls);
- If wall friction and viscoelasticity cannot be neglected, the same analysis holds with the exception that  $Z_c$  is a complex number.



$$P_f = \frac{P_m + Z_c Q_m}{2},$$

$$P_b = \frac{P_m - Z_c Q_m}{2},$$

$$Z_c = \rho \frac{c}{A}$$

# Exercises

1) Aorta wave speed?

$$\Delta t_1 = 0.056s$$

$$\Delta x_1 = 0.25m$$

2) Aorta  $Z_c$ ?

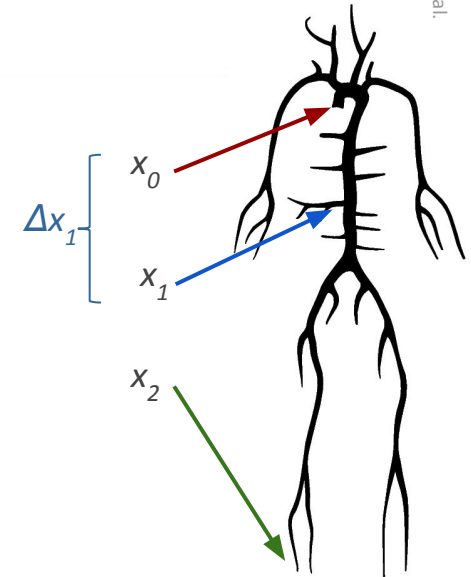
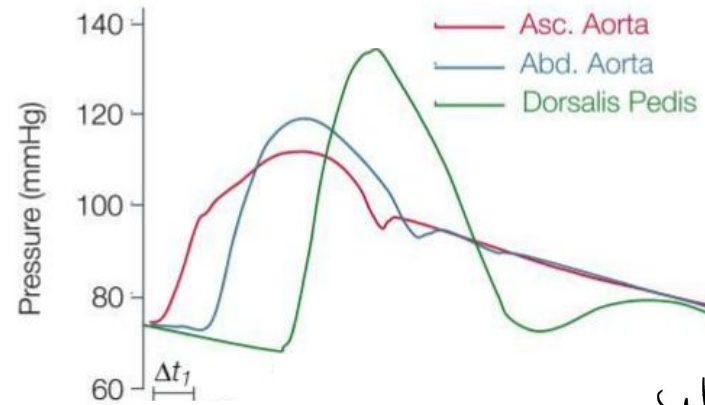
$$\rho = 1060 \text{ kg/m}^3, R_0 = 9.87mm$$

3) Aorta  $E$ ?

$$h = 0.82 \text{ mm}$$

4) Ascending aorta AI?

5) Dorsalis pedis AI?



# Summary

- Incident waves are generated by the pulsatile action of the heart
- Waves propagate at a finite speed depending on the arterial wall mechanical properties
- Waves are reflected by discontinuities in the arterial network
- Waveforms are the sum of incident and reflected waves
- Wave speed changes with age and can be measured with the foot-to-foot method
- Augmentation index is an hypertension metrics
- Waveform analysis is used to split forward and backward waves